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EFFECTS OF VARYING IN-PLANE FORCES ON VIBRATION OF ORTHOTROPIC RECTANGULAR PLATES RESTING ON PASTERNAK FOUNDATION

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ABSTRACT

Vibrational behavior of orthotropic rectangular plates resting on Pasternak foundation having two opposite edges(y = 0 and b) simply supported, with those edges subjected to linearly varying in-plane stresses σ_y = -N_0 (1-γx/a) h, where h is the plate thickness and the other two edges (x = 0 and a) may be clamped or simply supported has been discussed on the basis of classical plate theory. By assuming the transverse displacement (w) to vary as sin(p Л y/b), the governing partial differential equation of motion is reduced to fourth order ordinary differential equation in x with variable coefficients. Chebyshev collocation method has been used to obtain the first three modes of vibration for two different combinations of clamped and simply supported boundary conditions: clamped at x = 0 and clamped or simply supported at the edge x = a. Effect of in-plane force together with elastic foundation and other plate parameters such as orthotropy, aspect ratio on the natural frequencies of vibration is illustrated for the first three modes of vibration. Normalized displacements are presented for specified plates for all the three boundary conditions. Comparison of results with those published in literature demonstrates the computational efficiency of the method.

KEY WORDS: *Pasternak foundation; in-plane force; Chebyshev collocation method; aspect ratio.*

INTRODUCTION

The problem of buckling and frequency of flexural vibration of rectangular plates, which are subjected to edge loads in their mid-plane areas, has a vital role in the various engineering field such as shipbuilding, aircraft and automotive industries. A significant amount of work dealing with buckling and vibration of rectangular plates having uniformly distributed in-plane edge loadsisavailableintheliterature.However,platetypestructuralelementsareloadedbynonuniform in-plane loadings. The first variation from the uniform loading is one, which varies linearly (Fauconneau and Marangoni [1], Gorman [2] and Naguleswaran [3]); the other variations may be parabolic (Bert and Devarakonda [4]; Hu et al. [5]) or harmonic variations (Benoy [6]). Recently, Kang and Leissa [7] and Leissa and Kang [8] have analyzed the buckling and vibration of isotropic rectangular plates using Frobénius method i.e. exact solutions are obtained when two opposite edges are simply supported and these are subjected to linearly varying in-plane loading while the other two are either clamped (i.e. SS-C-SS-C plate) or free (i.e. SS-F-SS-F plate), respectively. Very recently, Wang et al. [9] employed differential quadrature method to obtain the numerical results for the buckling and vibration of isotropic SS-C-SS-C rectangular plate subjected to linearly varying in-plane stresses along the simply supported edges on the basis of classical plate theory. In the wide literature devoted to buckling and vibrational behaviour of plates, there is a lack of analysis on the buckling and vibration of orthotropic rectangular plates subjected to non-uniform in-planeloads.

In the past, the studies of vibrations of rectangular plates resting on Pasternak foundation have attracted various researchers due to their significant contribution in foundation engineering. Omurtag and Kadioglu [10] presented free vibration analysis of orthotropic Kirchoff rectangular plates resting on Pasternak foundation by using finite element method. Shen et al. [11] employed Rayleigh-Ritz method for free and forced vibration analysis of moderately thick isotropic rectangular plates with free edges resting on Pasternak foundation. Malekzadeh and Karami [12] analyzed the free vibration analysis of isotropic non-uniform thick rectangular plates resting on Pasternak foundation by using differential quadrature method. Leung and Zhu [13] obtained the transverse vibration of Mindlin plates of various geometries on two-parameter foundations using finite element method with trapezoidal *p*-elements.

The work presented in this paper aims to study the effect of Pasternak foundation on the transverse vibration of orthotropic rectangular plates subjected to linearly varying in-plane normal stresses on the basis of classical plate theory. The two opposite edges of the plate are assumed to be simply supported and these edges are subjected to linearly varying in-plane loads. The type of loading considered here gives rise to a differential equation with variable coefficients whose exact solution is not possible. A semi-analytical approach has been used. Accordingly the mode function in the direction of simply supported edges is represented by the trigonometric sine function. The governing partial differential equation of motion gets reduced to an ordinary differential equation, which has been solved numerically by using Chebysev collocation technique. In this model ―ORTHO1‖material has been taken as an example of a rectangular orthotropic material. The effect of in-plane force parameter, loading parameter, aspect ratio together with foundation parameters on the natural frequencies for the first three modes of vibration has beencomputed.

MATHEMATICAL FORMULATION

Consider a thin rectangular plate of length a, width b, uniform thickness h, density $\rho$and resting on two-parameter Pasternak foundation. The plate is referred to a system of rectangular Cartesian coordinates (x, y, z), the middle surface being z = 0 and the origin is at one of the corners of the plate. The x- and y- axes are taken along the principal directions of orthotropy and the axis of z is perpendicular to the xy-plane. The plate that is simply supported at y = 0 and b is taken to be under linearly varying in-plane stresses at these two edges. The other two edges i.e. $x = 0$ and a may be clamped or simply supported (Fig.1). The differential equation of motion governing the vibration of such plates is given by

$$D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + \rho h \frac{\partial^2 w}{\partial t^2} + k_f w - G_f \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - N_y \frac{\partial^2 w}{\partial y^2} = 0 , \tag{1}$$

where

$$N_y = \sigma_y / h = -N_0 (1 - \gamma x / a),$$

and N_0 is the intensity of compressive force at the edge $x = 0$, γ is the loading parameter.

For a harmonic solution, following Lévy approach i.e. the two edges $y = 0$ and $y = b$ are simply supported, the deflection w is assumed to be

$$w(x,y,t) = \bar{w}(x) \sin(p \pi y / b) e^{i \omega t} , \tag{2}$$

where p is a positive integer and ω is the frequency in radians.

Using equation (2) and introducing the non-dimensional variables

$$X = x/a, Y = y/b, \bar{h} = h/a, \quad W = \bar{w}/a, \tag{3}$$

equation (3) reduces to

$$A_0 W^{iv} + A_1 W''' + A_2 W'' + A_3 W' + A_4 W = 0, \tag{4}$$

$$A_0 = 1, \quad A_1 = 0, \quad A_2 = -2\lambda^2\left(\nu_y + \frac{\sqrt{E_x/E_y}}{1+\sqrt{\nu_x\nu_y}} - 12G/\bar{h}^3\right), \quad A_3 = 0,$$

where
$$A_4 = \lambda^4 E_x/E_y + 12(\lambda^2 G + K)/\bar{h}^3 - \lambda^2 N_0^*(1-\gamma X) - \Omega^2,$$
$$\lambda^2 = p^2\pi^2 a^2/b^2, K = ak_f(1-\nu_x\nu_y)/E_x, G = G_f(1-\nu_x\nu_y)/aE_x,$$
$$N_0^* = 12N_0(1-\nu_x\nu_y)/aE_x\bar{h}^3, \; \Omega^2 = 12\rho a^2\omega^2(1-\nu_x\nu_y)/E_x\bar{h}^2,$$

and primes denote differentiation with respect to X.

Equation (4) is a fourth order linear differential equation with variable coefficients of various parameters and its exact solution is not possible. An approximate solution of this eigen value problem subject to the boundary conditions at the edges X =0 and 1, is obtained by Chebysev collocation method.

METHOD OF SOLUTION: CHEBYSHEV COLLOCATION TECHNIQUE

By introducing a new independent variable $\xi=2X-1$, the range $0\le X\le 1$ is transformed to $-1\le\xi\le 1$, the applicability range of the technique. Equation (4) reduces to

$$B_0W^{iv} + B_1W''' + B_2W'' + B_3W' + B_4W = 0, \tag{5}$$

where, $B_i = 2^{4-i}A_i$, i = 0,1,2,3,4.

According to Chebyshev collocation technique Lal et al. [14], the highest order derivative is approximated as

$$\frac{d^4W}{d\xi^4} = \sum_{k=0}^{m-5} c_{k+5}T_k, \tag{6}$$

and its successive integrations lead to

$$W = c_1 + c_2 T_1 + c_3 T_1^1 + c_4 T_1^2 + \sum_{k=0}^{m-5} c_{k+5} T_k^4, \qquad (7)$$

where c_j (j=1,2,…,m) are unknown constants, T_k (k=0,1.2,…,m-5) are Chebyshev polynomials and T_k^j represents the j^{th} integral of T_k which are defined as

$$T_0^1 = T_1, \quad T_1^1 = (T_2 + T_0)/4,$$
$$T_j^1 = \int T_j \, d\xi = [T_{j+1}/(j+1) - T_{j-1}/(j-1)]/2, \quad j > 1,$$
$$T_j^i = \int T_j^{i-1} d\xi, \quad T_j = 2\xi T_{j-1} - T_{j-2}, \quad T_1 = \xi, \quad T_0 = 1.$$

Substitution of W and its derivatives in equation (5) gives an equation in terms of the T_j's and c_j's. The satisfaction of this resulting equation at (m-4) zeros of the Chebyshev polynomial T_{m-4} are given by

$$\xi_k = \cos\left(\frac{2k+1}{m-4}\frac{\pi}{2}\right), \; k = 0,1,2,\ldots,m-5, \qquad (8)$$

provides a set of (m-4) homogeneous equations in terms of the unknowns c_j(j = 1,2,…,m), which can be written in matrix form as

$$[B][C^*] = [0], \qquad (9)$$

where B and C^* are matrices of order (m-4)×m and m×1, respectively.

BOUNDARY CONDITIONS AND FREQUENCY EQUATION

The two sets of boundary conditions namely, C-C and C-S have been considered in which the first symbol represent the condition at edge X=0 and the second symbol at the edge X=1 and C, S stand for clamped and simply supported edge, respectively. The relations which should be satisfied at clamped and simply supported edge are

$W = \frac{dW}{dX} = 0$ and $W = \frac{d^2W}{dX^2} - \lambda^2 \frac{E^*}{E_x^*} W = 0$, respectively.

Applying the C-C boundary condition, one obtains a set of four homogeneous equations in terms of unknowns W_j (j =1,2,…,m). These equations together with field equation (9) give a complete set of m homogeneous equations in m unknowns which can be denoted by

$$\begin{bmatrix} B \\ B^{CC} \end{bmatrix} \left[W^* \right] = [0], \quad (10)$$

where B^{CC} is a matrix of order 4×m.

For a non-trivial solution of equation (10), the frequency determinant must vanish and hence,

$$\begin{vmatrix} B \\ B^{CC} \end{vmatrix} = 0. \quad (11)$$

Similarly for C-S plate, the frequency determinant can be written as

$$\begin{vmatrix} B \\ B^{CS} \end{vmatrix} = 0. \quad (12)$$

NUMERICAL RESULTS AND DISCUSSIONS

To investigate the effect of foundation on the frequency parameter Ω for the first three modes of vibration, the lowest three roots of the frequency equations (11-12) have been obtained using bisection method for various values of in-plane force parameter N_0^* (= ±70, ±50, ±30, 0), loading parameter γ(=0,0.5,1.0,1.5,2),aspectratioa/b(=0.5,1,0,1.5,2),foundationparameters:

Winkler foundation K(=0.0, 0.02, 0.04, 0.06) and Pasternak (two parameter) foundation$G$(=0.00, 0.02 ,0.004,0.006)for$p$=1.Theelasticconstantsfortheplatematerialaretakenas

$E_1 = 1\times10^{10}\,MPa$, $E_2 = 5\times10^{9}\,MPa$, $\nu_x = 0.2$, $\nu_y = 0.1$, given by Biancolini et al. [15](‗ORTHO1‘). The thickness h_0 at the edge $X = 0$ has been taken as 0.1.

To choose the appropriate number of collocation points *m*, convergence study was carried out for different sets of plate parameters. Convergence graph for C-C and C-S plates are presented in Figs. 2(a, b), respectively for *a*/*b*=1, N_0^*=30, γ=1, *K*=0.02, *G*=0.002. It is observed that four digit exactitude in values of frequency parameter Ω can be achieved by fixing *m* = 17.

The results are presented in Tables (1, 2) and Figures (3-7). Tables (1, 2) present the first three frequency parameter Ω for vibration of C-C and C-S plates, respectively for various values of in-plane force parameter N_0^*(= -50, -30, 0, 30 50), loading parameter γ(= 0, 0.5, 1, 1.5, 2), aspect ratio *a*/*b* (= 0.5, 1, 1.5, 2), and the foundation stiffness *K* = 0.0, 0.02, *G*= 0.00, 0.002. It is observed that the frequency parameter Ω for a C-C plate is higher than that for a C-S plate for the same set of values of plate parameters. Further, the frequency parameter Ω is found to increase with the increasing values of foundation parameters *K* and *G* whatever the values of other plate parameters. Frequencies for rectangular plate with aspect ratio *a/b* = 0.5 are smaller thanthatforasquareplate.Withtheincreasingvaluesofloadingparameterγ,thefrequency parameter Ω is found to increase for $N_0^* > 0$ and decrease for $N_0^* < 0$.

Figures 3(a-c) show the plots of frequency parameter Ω versus in-plane force parameter N_0^* for loading parameter γ= 0, 1, foundation parameters *K*= 0.02, *G*= 0.0, 0.002 and aspect ratio *a*/*b*=1 forC-CandC-Splatesforfundamental,secondandthirdmodesofvibration,respectively.Itis observed that the frequency parameter Ω decreases with the increasing values of in-plane force parameter N_0^* for γ=0 and 1 for both plates. It is noticed that the rate of decrease of Ω with N_0^* for C-S plate is higher than that for C-C plate. The rate of decrease for uniform loading (γ=0) is more than that for non-uniform (γ=1) loading in all the modes for C-C as well as C-S plates. The rate of decrease of Ω for plates resting on Winkler foundation is higher than those resting on Pasternak foundation. Also, the rate of decrease of Ω with N_0^* gets reduced with the increase in the number of modes.

Figures 4(a-c) depict the behaviour of frequency parameter Ω with loading parameter γ for N_0^* = -30, 30, *a*/*b* = 1 and three sets of foundation parameters namely *K* = 0.0, *G* = 0.0; *K* = 0.02, *G* = 0.0; *K* = 0.02, *G* = 0.002 for C-C and C-S plates vibrating in fundamental, second and third modes, respectively. It is observed that the frequency parameter Ω decreases with the increasing values of loading parameter γ for N_0^* = -30 while it increases for N_0^* = 30 in the absence as well as in the presence of foundation for both the plates. The frequency parameter Ω increases with the increase in the values of foundation parameters and the effect of shear stiffness parameter *G* is more pronounced as compared to Winkler stiffness parameter *K*. The effect of foundation parameters is more prominent for γ=0 as compared to γ=2 for N_0^*>0 and it is just the reverse when $N_0^* < 0$. The rate of decrease/increase of Ω with γ is higher for C-S plate as compared to C-C plate. Further, this rate of decrease / increase of Ω with γ decreases with the increasing number of modes.

The effect of aspect ratio *a*/*b* on the frequency parameter Ω forN^*_0= -20, 20, γ= 1, *K*=0.02 and *G*= 0.00, 0.002 have been shown in Figs. 7.5(a-c) for C-C and C-S plates vibrating in fundamental, second and third modes, respectively. It is observed that the frequency parameter Ω increases with the increasing values of aspect ratio *a*/*b* irrespective of other plate parameters. The rate of increase is more pronounced when *a*/*b*>1 as compared to *a*/*b*<1. This rate of increase of frequency parameter Ω with increasing values of aspect ratio *a*/*b* for C-S plate is higher than that for C-C plate keeping all other parameters fixed. Also, the rate of increase gets reduced in higher modes.

Figure 6(a) depicts the effect of Winkler foundation parameter *K* on frequency parameter Ω for N^*_0 = -20, 20, γ= 0, 1, 2, *G* = 0.002 and *a*/*b*=1 for both the plates vibrating in the fundamental mode. It is observed that the frequency parameter Ω increases with the increasing values of foundation parameter *K* whatever be the values of other plate parameters. The rate of increase of frequencyparameterΩwith*K*ispronouncedforC-SplateascomparedtoC-Cplate.Thisrate increases with the increase in the values of $\gamma$for N^*_0 = -20 and decreases for N^*_0 = 20. For the secondandthirdmodesofvibrationFigs.6(b,c),respectively,itisevidentthatthebehaviourof Ω with *K* is almost the same as that of first mode except that the rate of increase of Ω with*K* decreases with the increase in the number of modes.

Figures 7(a-c) depict the behaviour of frequency parameter Ω with shear stiffness parameter *G* for N^*_0 = -30, 30, γ=0, 1, 2, *K*=0.02 and *a*/*b*=1 for both plates vibrating in fundamental, second and third modes, respectively. It is found that the frequency parameter Ω increases with the increasing values of foundation parameter *G*. The effect of *G* is more pronounced for C-S plate

G	*K*	*a/b*	0.5	1

as compared to C-C plate. The rate of increase of Ω with *G* increases with the increase in the value of γ for $N_0^* <$ 030 while decreases for $N_0^* > 0$. Also, the rate of increase of Ω with *G* gets pronounced with the increase in the number of modes.

For a specified plate i.e. γ=1, *a*/*b*=1, N_0^*=30, *K*=0.02, *G*=0.002 three dimensional mode shapes for C-C and C-S plates are shown in Figs. 8 and 9, respectively.

A comparison of the results for homogeneous ($\mu = \beta = 0$), isotropic ($E_2 / E_1 = 1$) plate of uniform thickness ($\alpha = 0.0$) with other methods has been presented in Tables 3 for $\upsilon = 0.3$. Table 3 shows a comparison of our results for $p = 1$ with the exact values from Leissa [16] and approximate results obtained by Frobinius method (Jain and Soni [18]), Chebyshev collocation method (Lal et al. [14]) and quintic spline technique (Lal and Dhanpati [17]). Excellent agreement of results shows the computational accuracy as well as efficiency of present technique (Chebysev collocationmethod).

Table 1: Values of frequency parameter Ω for C-C plate vibrating in first three modes

G	K	a/b	0.5					1				
		N_0^* \ γ	0	0.5	1	1.5	2	0	0.5	1	1.5	2
Mode-I												
		-50	25.8808	25.2775	24.6584	24.0223	23.3679	34.8295	33.0055	31.0631	28.9784	26.7181
		-30	24.9092	24.5348	24.1542	23.7671	23.3732	31.8701	30.6848	29.4473	28.1509	26.7870
	0	0	23.3762	23.3762	23.3762	23.3762	23.3762	26.8257	26.8257	26.8257	26.8257	26.8257
		30	21.7354	22.1568	22.5699	22.9753	23.3732	20.5798	22.3029	23.8967	25.3856	26.7870
0		50	20.5689	21.3048	22.0151	22.7021	23.3679	15.0378	18.6854	21.7125	24.3517	26.7181
		-50	30.1632	29.6471	29.1210	28.5845	28.0367	38.1195	36.4605	34.7119	32.8595	30.8846
		-30	29.3338	29.0165	28.6954	28.3703	28.0412	35.4359	34.3737	33.2738	32.1321	30.9442
	0.02	0	28.0437	28.0437	28.0437	28.0437	28.0437	30.9777	30.9777	30.9777	30.9777	30.9777
		30	26.6913	27.0356	27.3752	27.7104	28.0412	25.7591	27.1555	28.4790	29.7393	30.9442
		50	25.7503	26.3419	26.9196	27.4843	28.0367	21.5902	24.2723	26.6728	28.8618	30.8846
		-50	35.4880	35.0504	34.6068	34.1568	33.7002	44.4691	43.0561	41.5877	40.0579	38.4594
		-30	34.7858	34.5186	34.2492	33.9774	33.7032	42.1914	41.3035	40.3934	39.4595	38.5001
0.002	0.02	0	33.7050	33.7050	33.7050	33.7050	33.7050	38.5230	38.5230	38.5230	38.5230	38.5230
		30	32.5884	32.8710	33.1510	33.4284	33.7032	34.4664	35.5225	36.5450	37.5368	38.5001
		50	31.8223	32.3029	32.7759	33.2416	33.7002	31.4729	33.3708	35.1576	36.8496	38.4594
Mode-II												
		-50	63.9572	63.7157	63.4736	63.2307	62.9871	70.6159	69.7383	68.8524	67.9577	67.0542
		-30	63.5702	63.4245	63.2786	63.1324	62.9860	69.2041	68.6677	68.1282	67.5854	67.0392
	0	0	62.9853	62.9853	62.9853	62.9853	62.9853	67.0307	67.0307	67.0307	67.0307	67.0307
		30	62.3950	62.5431	62.6910	62.8386	62.9860	64.7845	65.3538	65.9193	66.4811	67.0392
0		50	61.9982	62.2466	62.4942	62.7410	62.9871	63.2427	64.2122	65.1703	66.1175	67.0542
		-50	65.8067	65.5720	65.3367	65.1008	64.8643	72.2952	71.4383	70.5737	69.7012	68.8205
		-30	65.4307	65.2891	65.1474	65.0054	64.8632	70.9169	70.3936	69.8673	69.3381	68.8059
	0.02	0	64.8626	64.8626	64.8626	64.8626	64.8626	68.7977	68.7977	68.7977	68.7977	68.7977
		30	64.2894	64.4332	64.5768	64.7201	64.8632	66.6110	67.1649	67.7153	68.2622	68.8059
		50	63.9045	64.1454	64.3857	64.6253	64.8643	65.1125	66.0546	66.9863	67.9081	68.8205
		-50	74.0812	73.8728	73.6639	73.4546	73.2449	80.9781	80.2138	79.4440	78.6688	77.8878
		-30	73.7473	73.6218	73.4961	73.3702	73.2442	79.7500	79.2849	78.8178	78.3486	77.8774
0.002	0.02	0	73.2438	73.2438	73.2438	73.2438	73.2438	77.8715	77.8715	77.8715	77.8715	77.8715
		30	72.7367	72.8638	72.9908	73.1175	73.2442	75.9466	76.4327	76.9166	77.3981	77.8774
		50	72.3967	72.6094	72.8217	73.0336	73.2449	74.6357	75.4588	76.2750	77.0846	77.8878
Mode-III												
		-50	122.8378	122.7122	122.8381	122.4608	122.3350	128.6059	128.1257	127.6443	127.1618	126.6780
		-30	122.6367	122.5613	122.6369	122.4103	122.3347	127.8362	127.5464	127.2563	126.9657	126.6747
	0	0	122.3346	122.3346	122.3346	122.3346	122.3346	126.6728	126.6728	126.6728	126.6728	126.6728
		30	122.0317	122.1075	122.0318	122.2590	122.3347	125.4986	125.7933	126.0876	126.3814	126.6747
0		50	121.8293	121.9558	121.8297	122.2086	122.3350	124.7098	125.2037	125.6964	126.1878	126.6780
		-50	123.8108	123.6862	123.5616	123.4368	123.3120	129.5356	129.0589	128.5810	128.1020	127.6218
		-30	123.6114	123.5365	123.4616	123.3867	123.3117	128.7715	128.4838	128.1958	127.9073	127.6186
	0.02	0	123.3116	123.3116	123.3116	123.3116	123.3116	127.6166	127.6166	127.6166	127.6166	127.6166
		30	123.0111	123.0863	123.1615	123.2366	123.3117	126.4512	126.7437	127.0357	127.3273	127.6186
		50	122.8103	122.9359	123.0613	123.1867	123.3120	125.6683	126.1585	126.6475	127.1353	127.6218
		-50	133.2620	133.1463	133.0304	132.9146	132.7986	139.2278	138.7843	138.3399	137.8945	137.4488
		-30	133.0767	133.0072	132.9376	132.8681	132.7984	138.5171	138.2497	137.9820	137.7140	137.4456
0.002	0.02	0	132.7983	132.7983	132.7983	132.7983	132.7983	137.4441	137.4441	137.4441	137.4441	137.4441
		30	132.5193	132.5891	132.6589	132.7287	132.7984	136.3627	136.6340	136.9049	137.1754	137.4456
		50	132.3330	132.4495	132.5659	132.6823	132.7986	135.6370	136.0913	136.5446	136.9969	137.4488

		N_0^* \ γ	0	0.5	1	1.5	2	0	0.5	1	1.5	2
							Mode-I					
		-50	20.1045	19.2146	18.2791	17.2908	16.2398	30.6414	28.2805	25.6806	22.7604	19.3759
		-30	18.8373	18.2718	17.6874	17.0822	16.4538	27.2306	25.6550	23.9673	22.1417	20.1410
	0	0	16.7577	16.7577	16.7577	16.7577	16.7577	21.1049	21.1049	21.1049	21.1049	21.1049
		30	14.3805	15.0915	15.7694	16.4185	17.0421	12.2200	15.2388	17.7393	19.9175	21.8701
0		50	12.5479	13.8693	15.0725	16.1838	17.2212	51.4768	9.4790	15.0520	19.0279	22.2787
		-50	25.3809	24.6820	23.9610	23.2157	22.4439	34.3351	32.2457	29.9916	27.5324	24.8078
		-30	24.3894	23.9554	23.5127	23.0608	22.5993	31.3290	29.9697	28.5382	27.0233	25.4098
	0.02	0	22.8215	22.8215	22.8215	22.8215	22.8215	26.1805	26.1805	26.1805	26.1805	26.1805
		30	21.1376	21.6276	22.1060	22.5736	23.0311	19.7314	21.7307	23.5517	25.2331	26.8011
		50	19.9362	20.7932	21.6143	22.4035	23.1640	13.8541	18.1618	21.6001	24.5369	27.1356
		-50	31.2081	30.6531	30.0866	29.5083	28.9172	41.0292	39.3257	37.5323	35.6356	33.6181
		-30	30.4072	30.0667	29.7218	29.3725	29.0186	38.5487	37.4698	36.3542	35.1983	33.9988
0.002	0.02	0	29.1646	29.1646	29.1646	29.1646	29.1646	34.4951	34.4951	34.4951	34.4951	34.4951
		30	27.8667	28.2334	28.5949	28.9516	29.3035	29.8969	31.2330	32.5089	33.7315	34.9068
		50	26.9668	27.5949	28.2079	28.8067	29.3922	26.3901	28.8508	31.1020	33.1865	35.1339
							Mode-II					
		-50	52.6550	52.3493	52.0424	51.7341	51.4244	60.3060	59.2300	58.1402	57.0359	55.9160
		-30	52.1843	51.9994	51.8140	51.6281	51.4417	58.6466	57.9841	57.3163	56.6429	55.9637
	0	0	51.4702	51.4702	51.4702	51.4702	51.4702	56.0655	56.0655	56.0655	56.0655	56.0655
		30	50.7461	50.9357	51.1249	51.3135	51.5017	53.3596	54.0812	54.7958	55.5036	56.2049
0		50	50.2575	50.5764	50.8938	51.2098	51.5243	108.3285	52.7184	53.9383	55.1381	56.3191
		-50	54.8868	54.5935	54.2992	54.0038	53.7073	62.2641	61.2224	60.1688	59.1024	58.0224
		-30	54.4354	54.2581	54.0804	53.9023	53.7239	60.6583	60.0180	59.3730	58.7232	58.0688
	0.02	0	53.7512	53.7512	53.7512	53.7512	53.7512	58.1665	58.1665	58.1665	58.1665	58.1665
		30	53.0581	53.2395	53.4205	53.6011	53.7813	55.5630	56.2564	56.9437	57.6251	58.3009
		50	52.5910	52.8958	53.1994	53.5018	53.8029	53.7574	54.9475	56.1190	57.2732	58.4110
		-50	64.0219	63.7689	63.5152	63.2608	63.0057	71.6853	70.7778	69.8623	68.9387	68.0064
		-30	63.6353	63.4826	63.3298	63.1766	63.0232	70.2950	69.7406	69.1831	68.6226	68.0589
0.002	0.02	0	63.0510	63.051	63.0510	63.0510	63.0510	68.1564	68.1564	68.1564	68.1564	68.1564
		30	62.4612	62.6164	62.7714	62.9261	63.0805	65.9485	66.5361	67.1200	67.7003	68.2772
		50	62.0649	62.3251	62.5845	62.8432	63.1011	64.4346	65.4344	66.4234	67.4020	68.3706
							Mode-III					
		-50	106.4055	106.2576	106.1095	105.9614	105.8131	112.7920	112.2320	111.6704	111.1070	110.5419
		-30	106.1733	106.0844	105.9954	105.9064	105.8174	111.9135	111.5751	111.2361	110.8964	110.5561
	0	0	105.8241	105.8241	105.8241	105.8241	105.8241	110.5828	110.5828	110.5828	110.5828	110.5828
		30	105.4738	105.5633	105.6527	105.7421	105.8314	109.2358	109.5819	109.9272	110.2719	110.6160
0		50	105.2397	105.3890	105.5383	105.6875	105.8365	183.4040	108.9097	109.4890	110.0663	110.6417
		-50	107.5273	107.3810	107.2345	107.0879	106.9412	113.8509	113.2962	112.7398	112.1818	111.6221
		-30	107.2976	107.2096	107.1216	107.0335	106.9454	112.9807	112.6455	112.3097	111.9733	111.6362
	0.02	0	106.9521	106.9521	106.9521	106.9521	106.9521	111.6627	111.6627	111.6627	111.6627	111.6627
		30	106.6055	106.6940	106.7825	106.8709	106.9593	110.3289	110.6715	111.0135	111.3548	111.6955
		50	106.3738	106.5216	106.6693	106.8169	106.9643	109.4307	110.0060	110.5795	111.1512	111.7210
		-50	117.8006	117.6664	117.5321	117.3977	117.2632	124.3119	123.8020	123.2910	122.7785	122.2649
		-30	117.5909	117.5103	117.4296	117.3489	117.2681	123.5154	123.2077	122.8995	122.5908	122.2817
0.002	0.02	0	117.2757	117.2757	117.2757	117.2757	117.2757	122.3110	122.3110	122.3110	122.3110	122.3110
		30	116.9597	117.0408	117.1218	117.2028	117.2838	121.0945	121.4079	121.7209	122.0333	122.3458
		50	116.7486	116.8839	117.0191	117.1543	117.2893	120.2767	120.8024	121.3266	121.8494	122.3709

Table 2: Values of frequency parameter Ω for C-S plate vibrating in first three modes

Table 3: Comparison of frequency parameter Ω for isotropic (E_2/E_1=1), homogeneous (μ=β= 0.0)

rectangular plate of uniform thickness (α = 0.0) for p = 1, υ = 0.3.

Boundary Conditions		***K* = 0.0**				***K* = 0.01**			
	a/b	0.5		1.0		0.5		1.0	
	Mode / Ref.	I	II	I	II	I	II	I	II
C-C	Liessa [16]	—	—	28.946	69.320	—	—	—	—
	Lal et al.[14]	23.816	63.635	28.951	69.327	26.214	64.472	30.954	70.187
	Jain &Soni [18]	23.816	63.535	28.951	69.327	—	—	—	—
	Lal and Dhanpati [17]	23.820	63.603	28.950	69.380	26.219	64.539	30.953	70.239
	Present	**23.8132**	**63.5379**	**28.9503**	**69.3269**	**26.2139**	**64.4721**	**30.9540**	**70.1872**
C-S	Liessa [16]	—	—	23.646	58.641	—	—	—	—
	Lal et al.[14]	17.332	52.098	23.646	58.646	20.503	53.237	26.060	59.661
	Jain &Soni [18]	17.332	52.097	23.646	58.646	—	—	—	—
	Lal and Dhanpati [17]	17.335	52.150	23.647	58.688	20.506	53.288	26.061	59.702
	Present	**17.3315**	**52.09797**	**23.6363**	**58.6464**	**20.5032**	**53.2370**	**26.0602**	**59.6612**

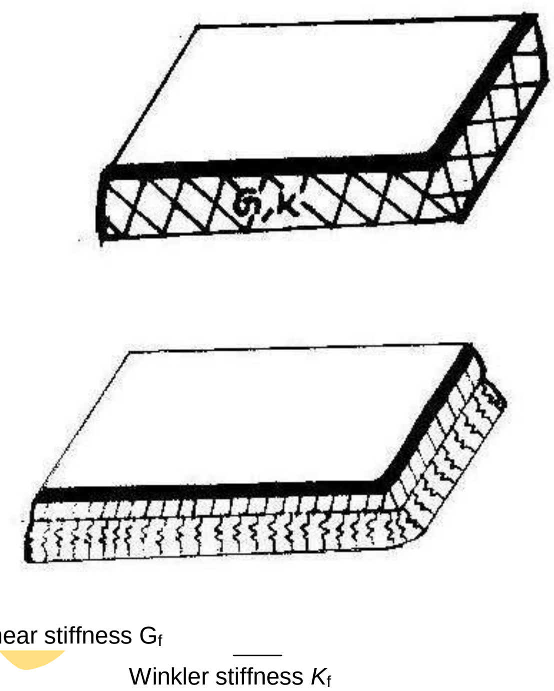


Fig. ***1: Rectangular plate under compressive force (γ = 1) and*** **on Pasternak foundation**

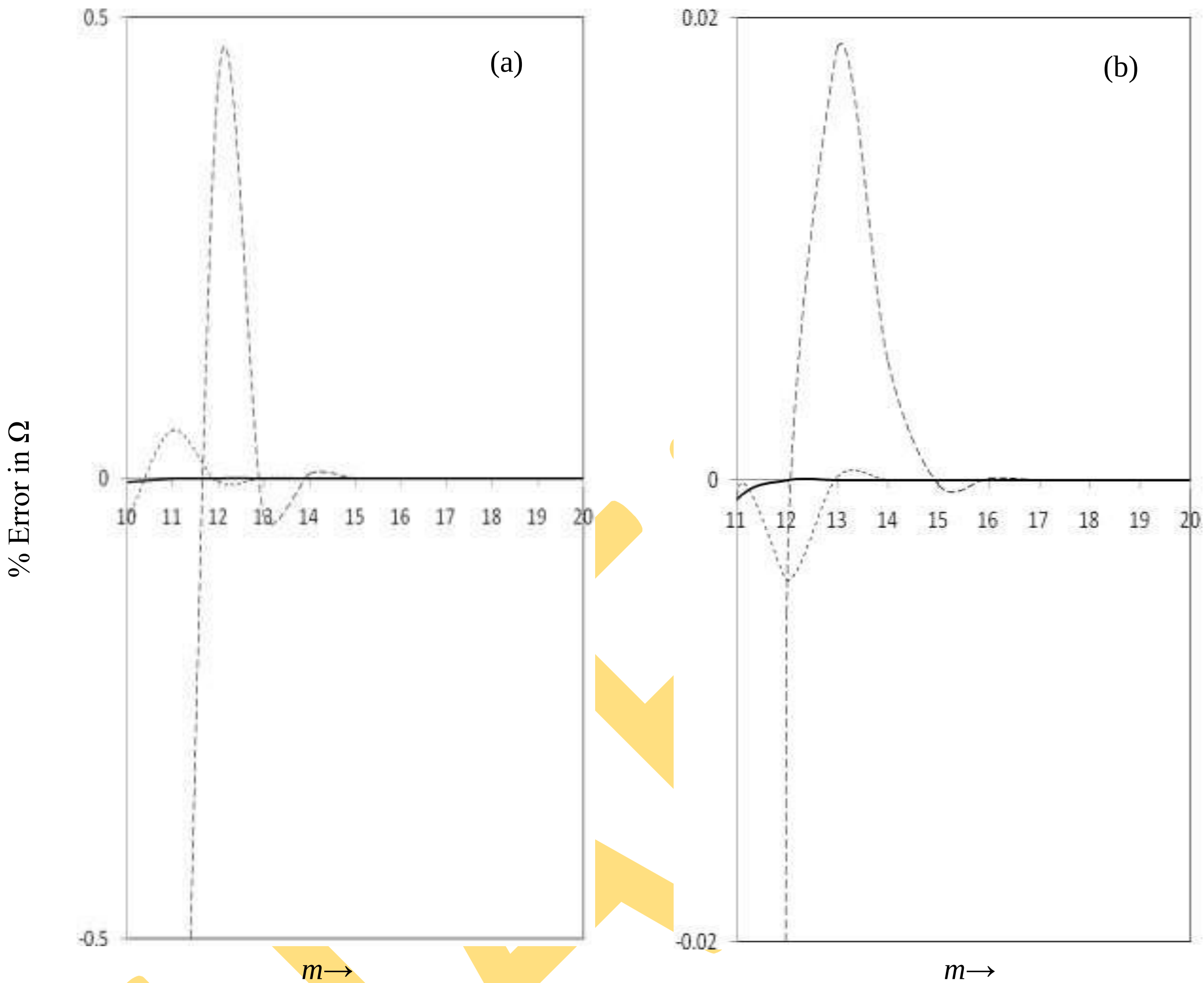


Fig. 2: Percentage error in frequency parameter Ω; (a) C-C plate and (b) C-S plate for N_0^*=30, γ=1, *a*/*b*=1, *K*=0.02, *G*=0.002.
———, fundamental; ...., second mode; – – – –, thirdmode.
% error = $[(\Omega_m - \Omega_{17}) / \Omega_{17}] \times 100$.

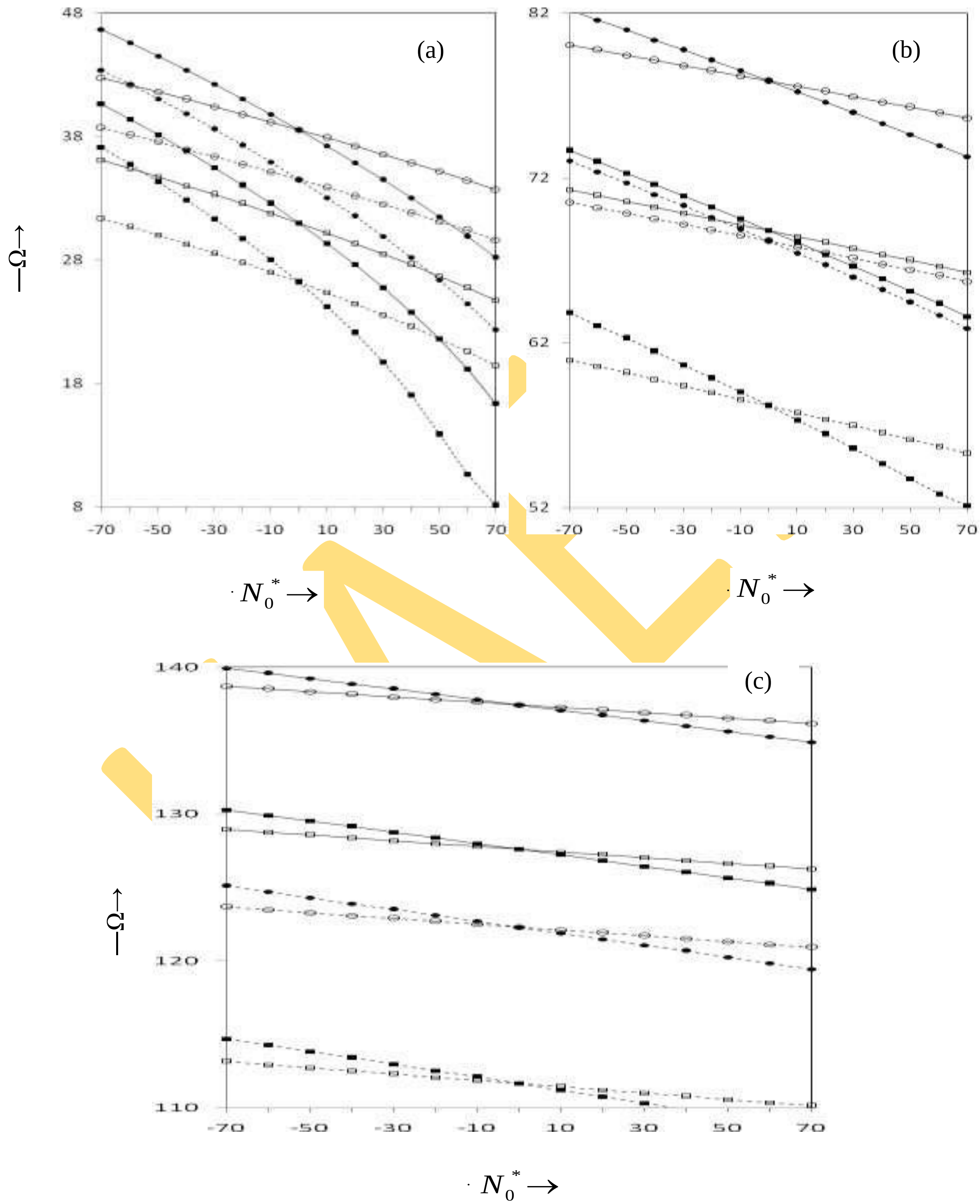


Fig..3: Frequency parameter Ω for C-C, C-S plates: (a) fundamental mode, (b) second mode, (c) third mode for *a/b*=1, *K*=0.002. ——, C-C; -------, C-S; ,■*G*=0.0, γ=0; □, *G*=0.0, γ=1; ●, *G*=0.002, γ=0; o, *G*=0.002, γ=1.

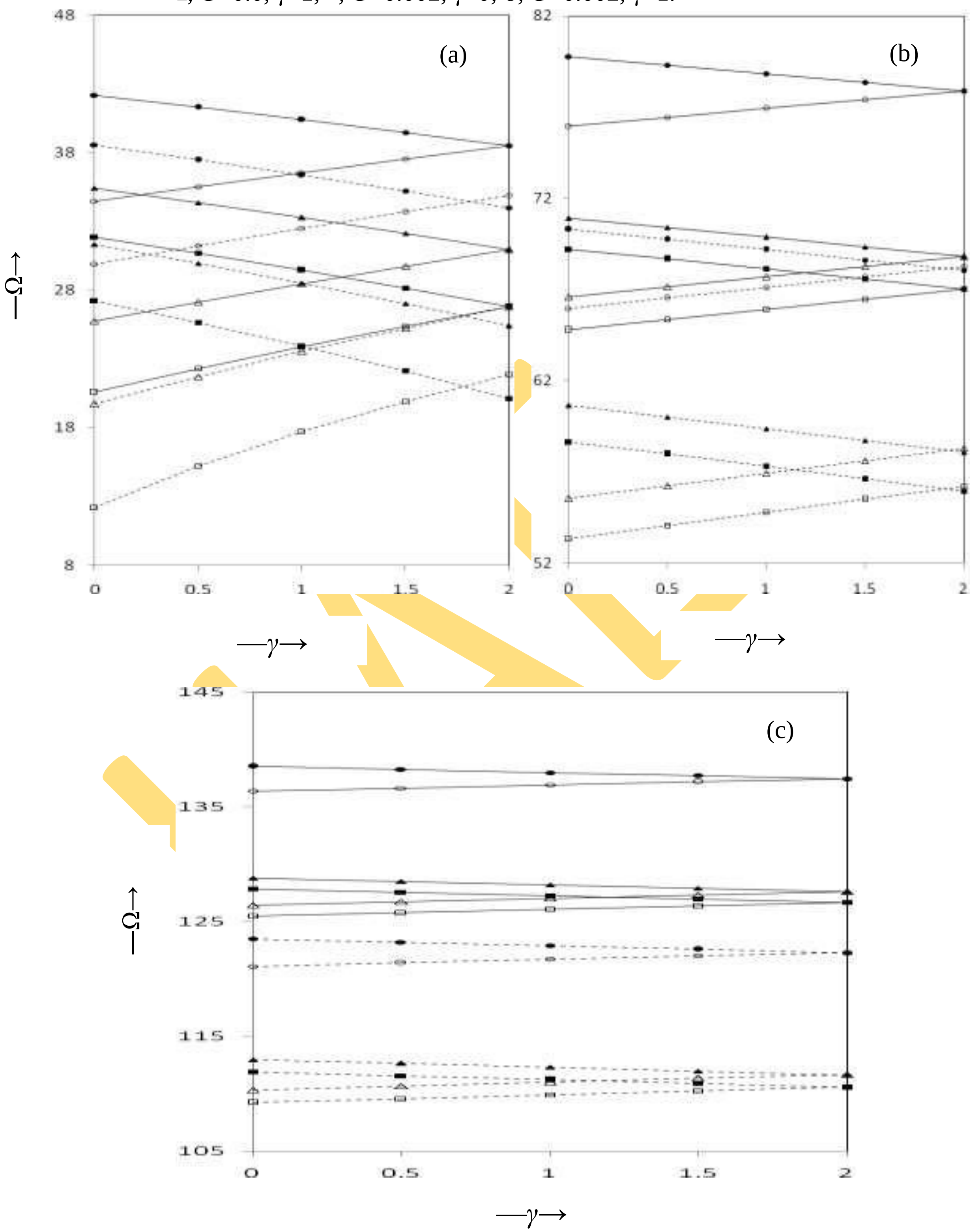


Fig. 4: Frequency parameter Ω for C-C, C-S plates: (a) fundamental mode, (b) second mode,

(c) third mode for *a*/*b*=1. ——, C-C; ------, C-S.■, □*K*=0.0, *G*=0.0; ▲, Δ, *K*=0.02,

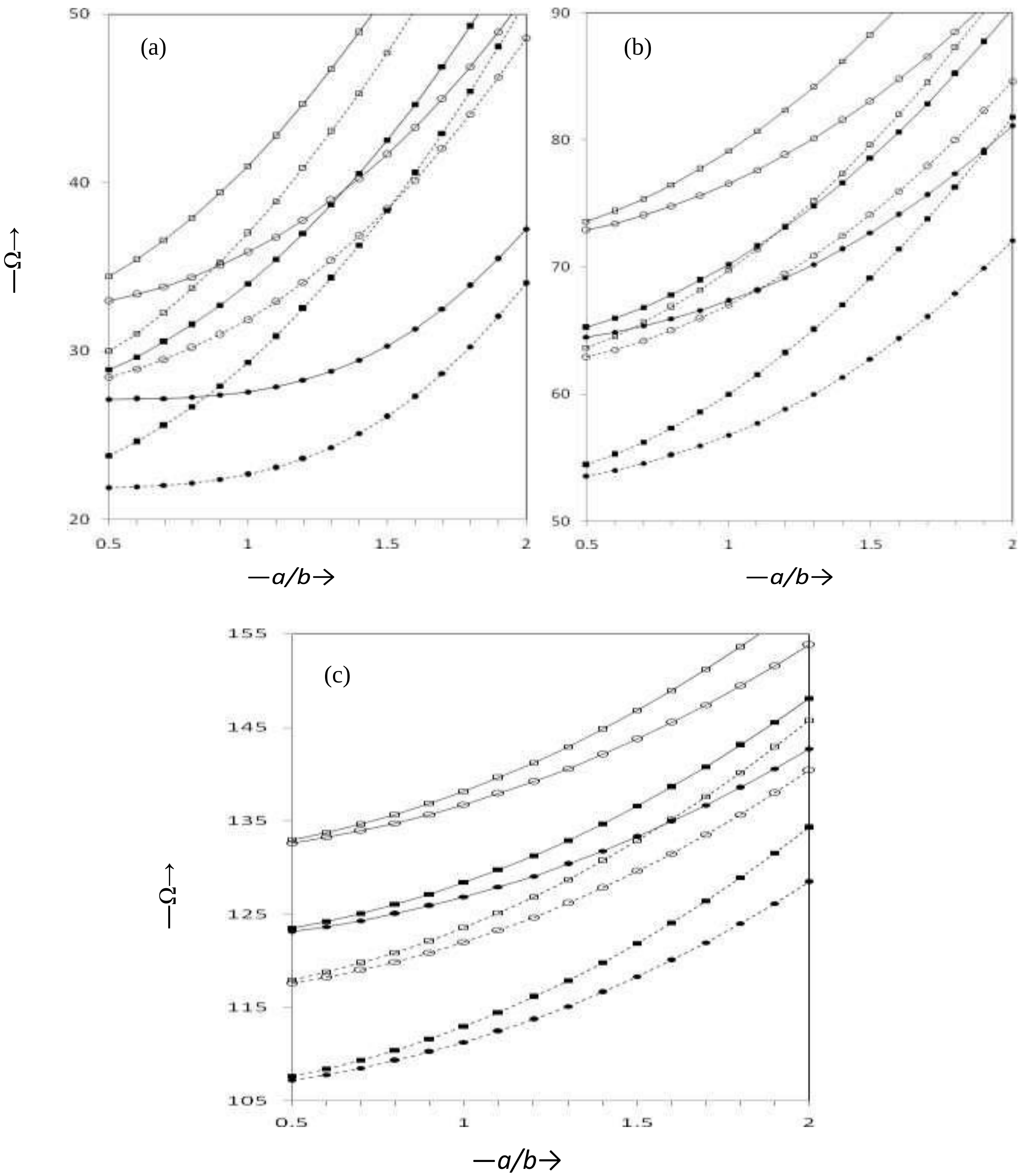
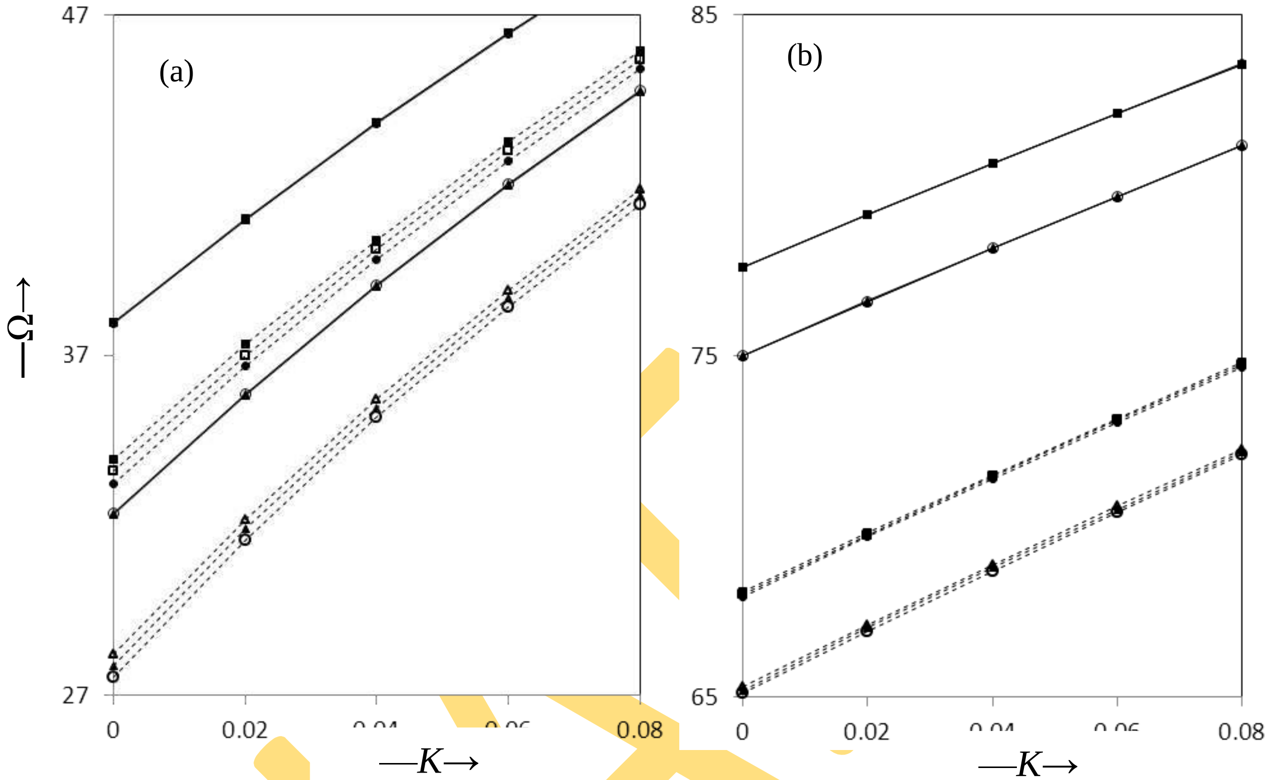


Fig. 5: Frequency parameter Ω for C-C, C-S plates: (a) first mode, (b) second mode,

(c) third mode forK=0.02,γ=1. ——, C-C;------,C-S; ■, N_0^* = -20, *G*=0.0; □, N_0^*= -20, *G*= 0.002; ●, N_0^*= 20, *G*= 0.0; o, N_0^*= 20, *G*= 0.002;



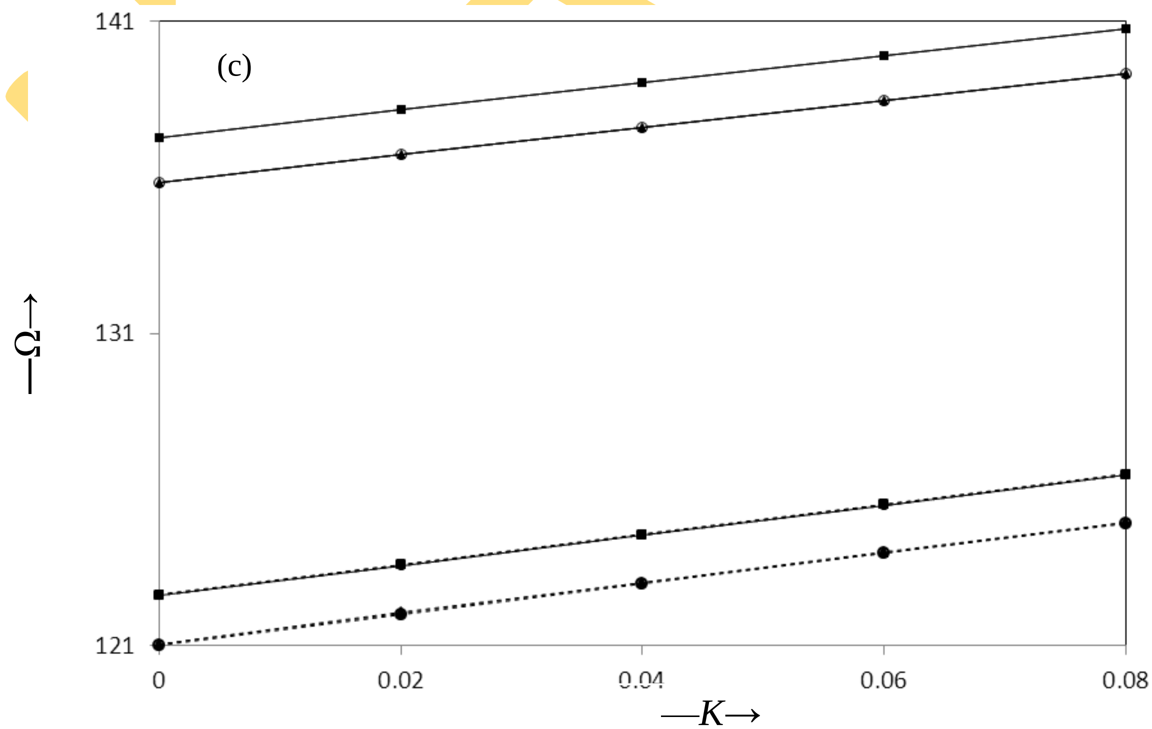
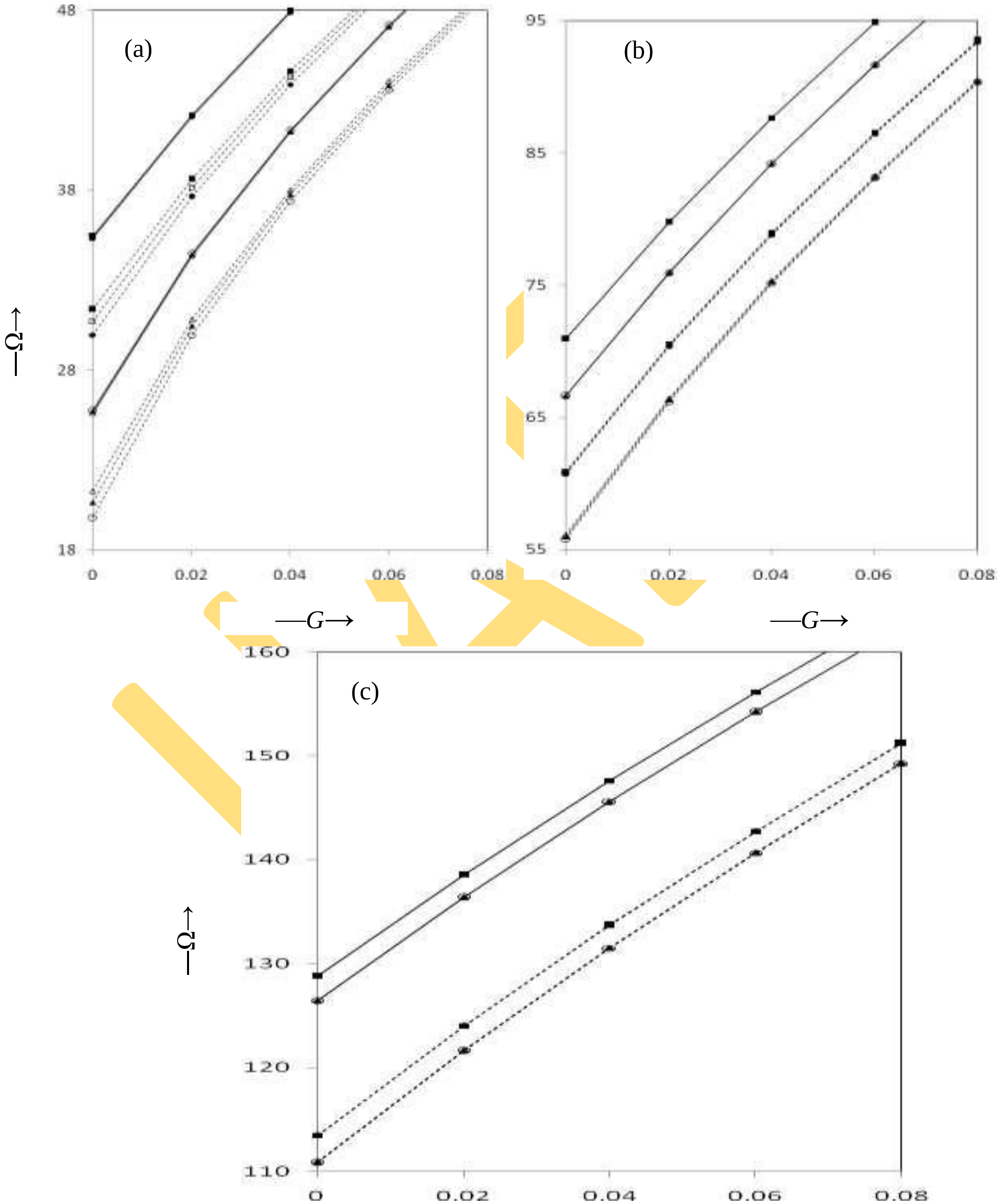
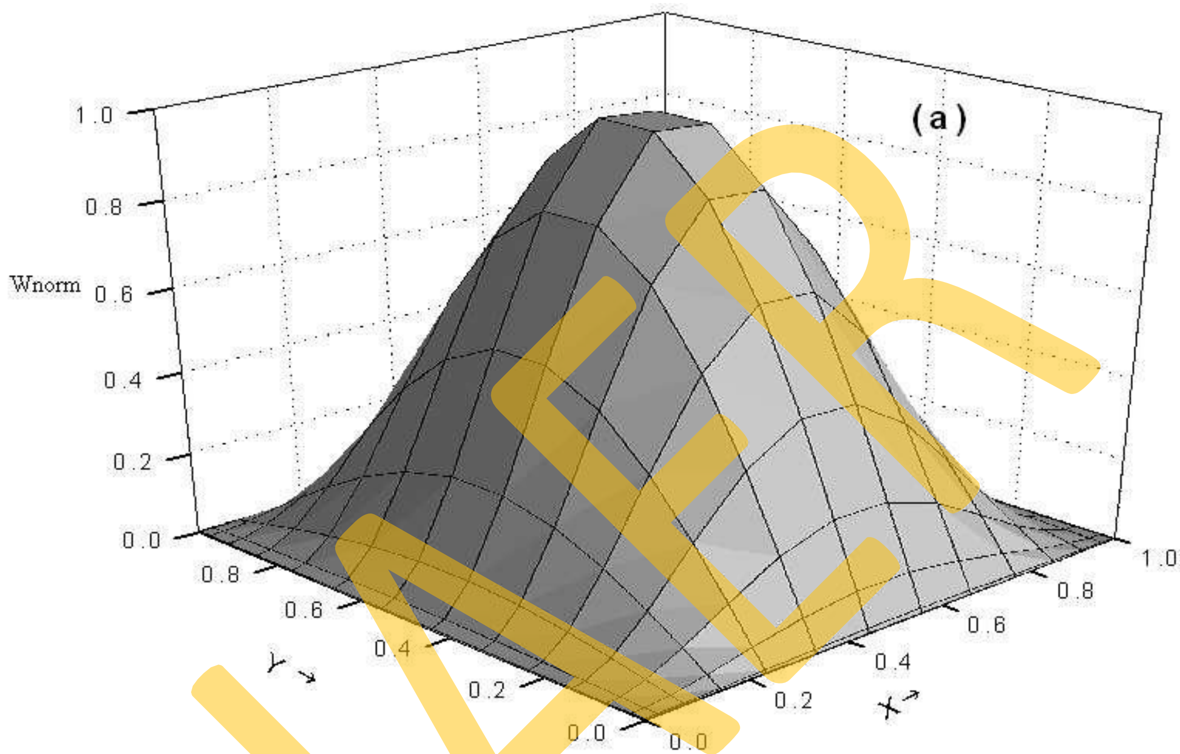


Fig. 6: Frequency parameter Ω for C-C, C-S plates: (a) first mode, (b) secondmode, (c) third mode for$G$=0.002, a/b=1. ——, C-C; ------, C-S; ■, γ=0, N_0^*=-20; □, γ=1, N_0^* = -20; •, o, γ=2, N_0^* = -20; o, γ=0, N_0^* = 20; ▲, γ=1, N_0^*=20; △, γ=2, N_0^*=20.



—*G*→

Fig.7: Frequency parameter Ω for C-C, C-S plates: (a) fundamental mode, (b) second mode, (c) third mode for K=0.02, a/b=1. ——, C-C; ------, C-S; ■, γ=0, N_0^*= -30; □, γ=1, N_0^*= -30; •, γ=2, N_0^*= -30; o, γ=0, N_0^* = 30; ▲, γ=1, N_0^*=30; △, γ=2, N_0^*=30.



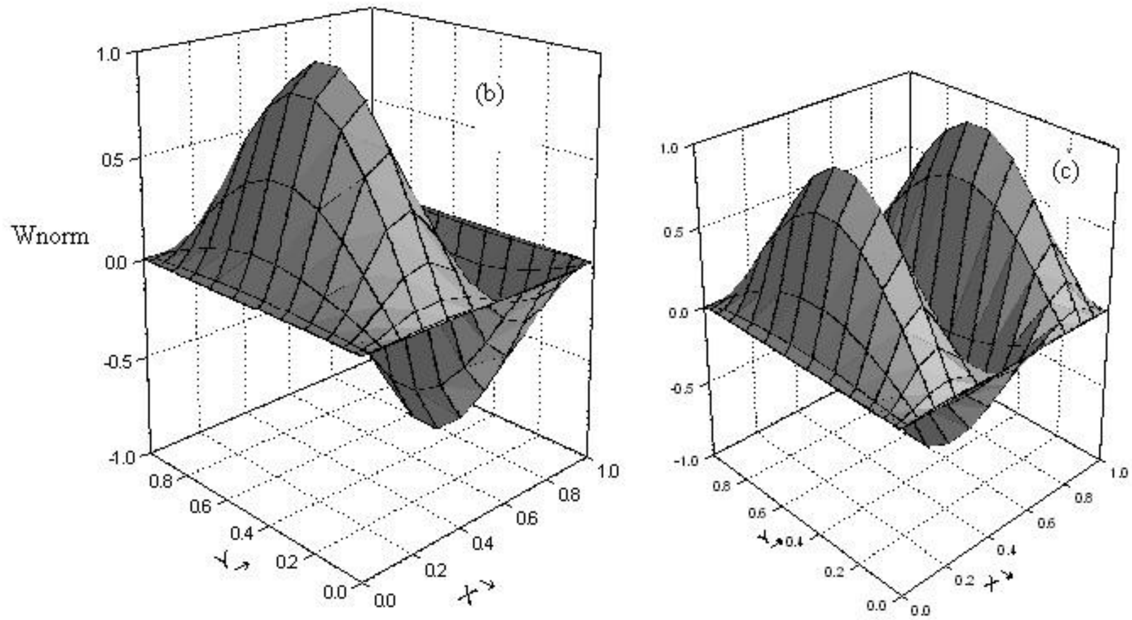


Fig. 8: Normalized displacement of C-C plate vibrating in (a) fundamental, (b) second and (c) third mode of vibration for $N_0^* = 20$, γ =1, K=0.02, G=0.002, a/b=1.

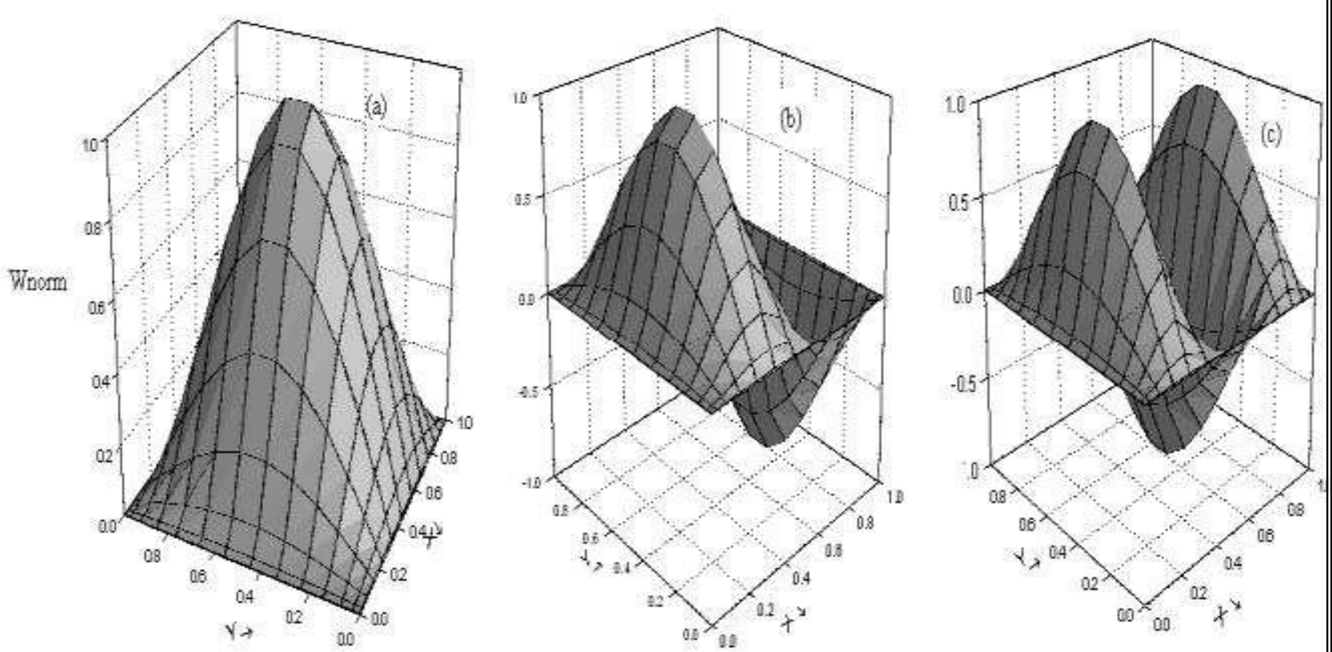


Fig. 9: Normalized displacement of C-S plate vibrating in (a) fundamental (b) second and (c) third mode; for $N_0^* = 20$, γ=1, K=0.02, G=0.002, a/b=1

REFERENCES

1. G. Fauconneau and R.D. Marangoni. Natural frequencies and elastic stability of a simply supported rectangular plate under linearly varying compressive loads. International Journal of Solids and Structures, 7, (1971),473-493.
2. D.J. Gorman. Accurate free vibration analysis of the completely free orthotropic rectangular plate by the method of superposition. Journal of Sound and Vibration, 165(3), (1993),409-420.
3. S. Naguleswaran. Transverse vibration of an uniform Euler-Bernoulli beam under linearly varying axial force. Journal of Sound and Vibration 275, (2004),47-57.
4. C.W. Bert and K.K. Devarakonda. Buckling of rectangular plates subjected to nonlinearly distributed in-plane loading. International Journal of Solids and Structures, 40, (2003),4097-4106.
5. H. Hu, A. Badir and A. Abatan . Buckling behavior of a graphite/epoxy composite plate under parabolic variation of axial loads. International Journal of Mechanical Sciences, 45, (2003),1135-1147.
6. M.B. Benoy. An energy solution for the buckling of rectangular plates under non-uniform in-plane loading. Aeronautical Journal, 73, (1969),974-977.

7. J.H. Kang and A.W. Leissa. Vibration and buckling of rectangular plates SS-F-SS-F rectangular plates loaded by in-plane moments. International Journal of Structural Stability and Dynamics, 1(4), (2001),527-543.

8. A.W. Leissa and J.H. Kang. Exact solution for vibration and buckling of an SS-C-SS-C rectangular plate loaded by linearly varying in plane stresses. International Journal of Mechanical Sciences 44, (2002),1925-1945.

9. X. Wang, L. Gan and Y. Wang. A differential quadrature analysis of vibration and buckling of an SS-C-SS-C rectangular plate loaded by linearly varying in-plane stress. Journal of Sound and Vibration, 298, (2006),420-431.

10. M.H. Omurtag and F. Kadioglu. Free vibration analysis of orthotropic plates resting on Pasternak foundation by mixed finite element formulation, Computers and Structures, 67(4), (1998),253-265.

11. H.S. Shen, J. Yang and L. Zhang. Free and forced vibration of Reissner-Mindlin plates with free edges resting on elastic foundations. Journal of Sound and Vibration, 244(2), (2001),299-320.

12. P. Malekzadeh and G. Karami. Vibration of non-uniform thick plates on elastic foundation by differential method. Engineering Structures 26, (2004).1473-1482.

13. A.V.T. Leung and B. Zhu. Transverse vibration of Mindlin plates on two-parameter foundation by analytical trapezoidal p-elements. Journal of Engineering Mechanics, 131(11), (2005),1140-1145.

14. R. Lal, U.S. Gupta and C. Goel. Chebyshev polynomials in the study of transverse vibrations of non-uniform rectangular orthotropic plates. The Shock and Vibration Digest, 33(2), (2001),103-112.

15. M.E. Biancolini, C. Brutti and L. Reccia. Approximate solution for free vibration of thin orthotropic rectangular plates. Journal of Sound and Vibration 288, (2005),321-344.

16. A.W. Leissa. Vibration of plates, NASA SP-160. Washington, DC: Government Printing Office,1969.

17. R. Lal and Dhanpati. Transverse vibrations of non-homogeneous orthotropic rectangular plates of variable thickness: A spline technique. Journal of Sound and Vibration, 306(1-2), (2007),203-214.

18. R.K. Jain and S.R. Soni. Free vibration of rectangular plates of parabolically varying thickness. Ind. Jl. of Pure and Applied Mathematics 4(3), (1973),267-277.